Measuring gravity at short distances
and other fun tricks
with levitated microspheres

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Image credit: Delia Gratta
My meandering thread.

- Measuring gravity at short distance, new physics opportunities
- The physics of optically levitated dielectric microspheres
- (non-gravity) results in particle physics
- New technical developments
- Towards a gravity measurement in the 1 - 50µm range

The goal is to convince you that we are developing a truly wonderful new technique with many exciting applications, not just in the area of short distance gravity. Indeed there now several groups active in this field, worldwide.
Gravity is:
- the most evident
- the weakest
- the least well known interaction in Nature

Most of the empirical features of gravity and differences in phenomenology from the other interactions can be understood in terms of the parameters above.

In addition there is no such thing as “antigravity”, so gravity cannot be shielded, which explains why this weakest force is so evident and, e.g. keeps the solar system together.
The first laboratory experiment on gravity

Apparatus by Rev. John Mitchell, used by Henry Cavendish to “Determine the Density of the Earth”.

H. Cavendish, Phil. Trans. Royal Soc. London (part II) 88, p469-526 (21 Jun 1798, 220 years ago!)
Cavendish’s measurement, in terms of $G$, gives

$$G = (6.74 \pm 0.04) \cdot 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \sim 0.6\%, \quad (1798)$$

Current measurements have 11 ppm uncertainty, but there is a few sigma disagreement between the two most recent ones (Li et al., Nature 560 (2018) 582)

At the same time we know
- the QED coupling constant, $\alpha$, to 0.23ppb
- the weak coupling constant, $G_F$, to 0.5ppm
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However, since we do not know how to calculate $G$ from other quantities in physics, we do not expect to find new physics in the absolute value of $G$.

More interesting is to test if there are deviations from the $1/R^2$ law for gravity.
It is customary to express potential deviations from the $1/R^2$ law by modifying the potential with a Yukawa term, obtaining:

$$V(R) = G \frac{M_1 M_2}{R} \left(1 + \alpha e^{-R/\lambda}\right)$$

$\alpha$: magnitude of the effect
$\lambda$: scale of the effect
What do we empirically know
What do we empirically know
What do we empirically know

- Lunar ranging
- Planetary Motion

Today’s primary task

In addition, there are important theoretical reasons to suspect that deviation from $1/R^2$ may actually arise naturally and be more than just plausible.

Gravity is a notoriously rebellious interaction. We do not have a good framework to treat the theory of gravity in a quantum-mechanical context.

And gravity at ordinary energies/distances is so much weaker than any of the other fundamental interactions.

Why? Are those issues related to each other?

Will the solutions of these puzzles simultaneously solve other modern puzzles in physics, such as those of Dark Matter and Dark Energy.
As an example, several authors have suggested that the dimensionality of the physical universe may at the root of some of these problems.

Many theories naturally include more than the ordinary 3 space dimensions. Since the ordinary physical space is clearly 3-dimensional, it is often assumed that the dimensions in excess of 3 are somehow curled up at very small scale, so that they have no effect at larger scales.

E.g. here the field scales as $1/R$ for $R<<a$ and as a constant for $R>>a$.

Some versions of this scenario substantially reduce the scale gap from electroweak physics to gravity, making gravity stronger at energy scales that are not as extreme as would result from a plain $1/R^2$ trend.

But, also, any new, long range force related to intrinsic properties (e.g. baryon number) may appear as a modification to Newtonian gravity.
Experimental challenges

• Since $F = G \frac{M_1 M_1}{R^2} = G \frac{\rho_1 V_1 \rho_2 V_2}{R^2}$

for materials we have access to (no Neutron Stars here!)

$\rho_1 \sim \rho_2 < 20 \text{ g/cm}^3$, there is no silver bullet.

In addition the volume $V \sim R^3$, so $F \sim G \frac{\rho^2 R^6}{R^2}$ and it is clear that measurements at short distance become exceedingly difficult.

Often the measured quantity is the acceleration of the test mass, $a \sim G \frac{\rho R^3}{R^2} \sim G \rho R$, but as $R$ decreases this is still challenging.

• At distances <100µm even neutral matter results in residual E&M interaction that are a dangerous background for these measurements.
Most inverse-square law measurements done with wonderfully sophisticated versions of Cavendish’s setup.

As distances become shorter this approach becomes clumsy and substantial efforts have to do with “artificial” issues (e.g. how to machine a 5 cm diameter disk flat to µm level…).

In recent times, some new measurements have been made using AFM techniques (but, still, this uses a mechanical spring).

Sketch of the EotWash apparatus from the University of Washington in Seattle
D. J. Kapner et al., PRL 98 (2007) 021101

Sketch of the custom cryogenic AFM apparatus from Kapitulnik’s group at Stanford
J. Chiaverini et al., PRL 90 (2003) 151101
The current experimental situation on the α-λ plane

Note: The ideal probe for such a measurement is the neutron (charge radius is ~1fm instead of ~1nm). For the same reasons the manipulation of neutrons is hard and results are only interesting at ultra short distance (where there no other option).
The current experimental situation on the $\alpha$-$\lambda$ plane

Excluded by experiments

The ideal probe for such a measurement is the neutron (charge radius is $\sim 1\text{fm}$ instead of $\sim 1\text{nm}$). For the same reasons the manipulation of neutrons is hard and results are only interesting at ultra short distance (where there no other option)

Lots of phase-space for new physics to explore

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U Chicago, Nov 15, 2018

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Given the small strength of gravity we need to measure really small forces
...some orders of magnitude of more or less familiar forces (weights)

A bathroom scale resolves $\sim 1 \, N$

A dust mite $10^{-7} \, N$

Virus $10^{-19} \, N$

Carbon atom $10^{-25} \, N$

90 kg $\sim 900 \, N$

Conventional AFM measures $10^{-12} \, N$

Specialized, cryogenic setups can result in $\sim 10^{-16} \, N/\sqrt{\text{Hz}}$ noise floors

E. coli $10^{-14} \, N$
A “new and old” technique to explore the short distance behavior

Optical Levitation by Radiation Pressure

A. Ashkin and J. M. Dziedzic

Bell Telephone Laboratories, Holmdel, New Jersey 07733

(Received 14 June 1971; in final form 13 August 1971)

The stable levitation of small transparent glass spheres by the forces of radiation pressure has been demonstrated experimentally in air and vacuum down to pressures ~1 Torr. A single vertically directed focused TEM_00-mode cw laser beam of ~250 mW is sufficient to support stably a ~50-μ glass sphere. The restoring forces acting on a particle trapped in an optical potential well were probed optically by a second laser beam. At low pressures, effects arising from residual radiometric forces were seen. Possible applications are mentioned.
Micro and nano-spheres are commercially available from different dielectric materials with diameters between 10 nm to 10µm

We use silica microspheres with 4.8µm diameter.

Different materials may also have interesting properties: to be tested later.
Microspheres optically trapped in vacuum make superb force sensors

- In high vacuum can cool the force sensor (μsphere) while everything else is at room temperature.
- Thermal and vibrational noise from mechanical support minimized.
- Trap parameters can be changed instantaneously.
- Control of optical potential and motion in all 3 DOF: great flexibility.
- Extremely low dissipation is possible: $Q \sim 10^{12}$ at $10^{-10}$ mbar.
- The noise quantum limit should be reachable (but it’s not required for the first gravity measurements).
- Microspheres are really isolated (in particular electrically).
- Unexplored: much risk and many opportunities!
- Many applications to other areas.

Image credit: Delia Gratta
Trap loading

- Microspheres are launched from bottom surface of quartz cantilever in a few torr atmosphere
- Pull-off forces of ~100 nN require accelerations ~$10^6$ m/s²
- Bottom coverslip protects lens and is retracted after trapping
- Once a µsphere is trapped, the buffer gas is slowly removed and the feedback cooling turned on.
Excellent charge control is obviously essential.

→ “funnels” shield the lenses and serve as electrodes for force calibration and more.

6 “funnels” can be independently biased.

Trapped μsphere

2.5 mm
As loaded in the trap, μspheres are usually charged (~500e)

⇒ Their charge state can be changed at leisure (in both directions), using a UV light source

The charge state can then be measured by applying an RF potential to a pair of electrodes

Quantized charge

“0 charge” with increased 500 V RF amplitude.
How close to 0 is “0 charge”?

There are small residuals but the response is not consistent with an effective charge.


The largest residual can be conservatively used as a limit to particles with a “milliclange” bound into/onto the μspheres.


Kim et al., PRL 99 (2007) 161804
Feedback cooling + high quality optics and a new single beam trap: $\Rightarrow$ essential for low noise + to bring objects close to the $\mu$sphere


The quadrant photodetector (QPD) is a very rudimentary tool for this purpose. Better tools are being worked on.
Feedback cooling is required below 0.1 mtorr and, at sufficiently low pressure can “cool” the C.O.M. of the μsphere to mK effective temperature (in a room temperature apparatus).

*Trapping lifetime is unlimited.*

Still, >2 orders of magnitude away from the quantum limit

⇒ Plenty of room for improvement

At least initially, reaching the quantum regime of the shot noise is not required, because the sensitivity is dominated by background forces.

Yet, we expect to eventually reach the regime dominated by quantum effects.
The current acceleration noise is among the best in the field.

<table>
<thead>
<tr>
<th>R (µm)</th>
<th>$f_{\text{trap}}$ (kHz)</th>
<th>$\sigma_a$ (ms$^{-1}$/√Hz)</th>
<th>$f_1$-$f_2$ (kHz)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>0.25</td>
<td>$7.5 \cdot 10^{-5}$</td>
<td>0.01 - 0.1</td>
<td>PRA 97 (2018) 013842</td>
</tr>
<tr>
<td>1.5</td>
<td>9.1</td>
<td>$4.6 \cdot 10^{-4}$</td>
<td>1 - 10</td>
<td>Nat. Phys. 7 (2011) 527</td>
</tr>
<tr>
<td>1.5</td>
<td>1.0</td>
<td>$7.7 \cdot 10^{-3}$</td>
<td>0.01 - 1</td>
<td>PRA 91 (2015) 051805(R)</td>
</tr>
<tr>
<td>0.15</td>
<td>2.8</td>
<td>$5.7 \cdot 10^{-2}$</td>
<td>0.01 - 3</td>
<td>PRA 93 (2016) 053801</td>
</tr>
</tbody>
</table>
There are specific challenges in moving objects very close to the microsphere:

- Perturbation of the optical mode of the trapping beam
- Residual electrostatic forces (typically gradients interacting with microsphere dipole moments)
- Casimir forces (in principle those should be calculable)

Distances down to a few µm have been stably achieved.
Reasonable Gaussian mode and suppression of scattered light from interferometry (still, at present, this is the main limitation and will be improved by better optics with a new trap and a pixelated photodetector)

A priori model from beam parameters (only free parameters: normalization and centering)
For a static attractor, the noise on the measurement is unaffected (and very small) -- even for a charged microsphere (~500e⁻)

Closest approach for the plot is 15μm, substantially smaller distances are possible, for now with larger noise
The force sensitivity is linear over several orders of magnitude, so that the microsphere can be used to sense different interactions, with

- state of the art noise
- good linearity
- 3D vector measurement
- relatively large sensitive volume
  
  \[ (80 \times 80 \times 80 \mu m^3, \text{limited by the range of the current positioning stage}) \]

Can’t get as close as AFM, but noise is similar to cryo AFM and this is a fully 3D measurements in a large volume of space
As a demo, full 3D vector mapping of the electric field of a biased attractor (100mV) --here compared to a FEA model

C.Blakemore et al., arXiv:1810.05779 (Oct 2018)
Similar results in G.Winstone et al., arXiv:1712.01426 (Dec 2017)
This can be used to map the patch potential distribution on the 200nm thick gold coating

Now the microsphere is still charged with $\sim 500e^{-}$ but the gold surface is grounded and scanned along $y$ at various $x$, $z$ positions.

Model is calculated by FEA starting from Kelvin probe microscopy of a similar surface
Robertson et al., Class. Quant. Grav. 23 (2006) 2665

The fit returns rms patch sizes 0.8$\mu$m in $x$ and 0.7$\mu$m in $y$ assuming $V_{\text{patch}} \sim 100$mV
Even with the initial sensitivity, the small size of microspheres allows the measurement of phenomena not accessible to macroscopic experiments.

This is the case of some models of Dark Energy assuming a field coupling to mass, with an energy scale $\Lambda \approx 2.4 \text{meV}$, corresponding to a length scale $\hbar c/\Lambda \approx 80 \mu m$.

In some cases the resulting forces can be substantially larger than gravity but they would evade detection by being screened by the matter density. So, at cosmological distance, the small matter density results in the manifestation of Dark Energy and yet lab measurements (with large densities around) have difficulties to detecting the field.

Fair to say that not everybody finds this Chameleon model compelling; in addition it is unclear how the non-linearity extrapolate to the regime that the experiment can test.
Permanent and induced electric dipoles in the microsphere are a potential background because of contact potentials in the cantilever.

Neutral microspheres contain $\sim 10^{14}$ electric charges and interact primarily as dipoles:

$$\vec{F} = (\vec{p} \cdot \nabla) \vec{E} \quad \Rightarrow \quad F_z \approx (p_0 + \alpha E_z) \frac{\partial E_z}{\partial z}$$

FEM calculation of electric potential:

Force for permanent and induced dipole:

Centered in $y$
This background is measured by deliberately biasing the cantilever 1 to 5V and sweeping its position in and out

- Fits to distance dependence allow determination of permanent and induced dipole moments
- For the microspheres used, most of the residual interaction is due to permanent dipole moments

\[ F \propto \frac{\partial E_z}{\partial z} \]

Microsphere response vs. distance:

<table>
<thead>
<tr>
<th>Microsphere</th>
<th>( p_0z ) [e ( \mu )m]</th>
<th>( \alpha/\alpha_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>151 ± 6</td>
<td>0.21 ± 0.13</td>
</tr>
<tr>
<td>#2</td>
<td>89 ± 10</td>
<td>0.00 ± 0.33</td>
</tr>
<tr>
<td>#3</td>
<td>192 ± 30</td>
<td>0.25 ± 0.14</td>
</tr>
</tbody>
</table>

Fits to dipole response:

Polarizability, \( \alpha \), measured relative to:

\[ \alpha_0 = 3\varepsilon_0 \left( \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \right) \left( \frac{4}{3}\pi r^3 \right) \]

for \( \varepsilon_r = 3, \ r = 2.5 \ \mu m \)

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Now repeat without bias on the cantilever to extract the Chameleon result

Atom interferometry is nominally more sensitive (M.Jaffe et al. Nature Physics 13 (2017) 938)
Spinning trapped microspheres

It has been demonstrated by others that birefringent microspheres can be spun up by applying a torque from a circularly polarized light beam. e.g. Y. Arita et al., Anal. Chem. 83 (2011) 8855

This technique can reach extremely high angular velocities (at the point of making the microspheres explode), but it does not allow for the control of the angular velocity.

If the microsphere has an electric dipole moment, then a torque can be applied by a rotating external electric field. We can apply this through the 4 electrodes in the horizontal plane.

The rotation is read out using the small residual birefringence that the silica microspheres apparently have.

As expected, the signal is at 2x the rotation frequency
Spindown rate depends on the pressure, as expected.

Assuming drag in $N_2$, $\tau$ would result in a pressure $10^{-5}$ mbar*, consistent with the measurements from vacuum gauges in system (vacuum gauges read the pressure outside the trap “cube”)

*After A.Cavalleri et al.,
It is easy to measure the phase lag between the drive signal and the spinning readout.

As expected, the phase lag is a function of the pressure (i.e., the gas drag).

When the phase lag reaches $\pi/2$, the microsphere unlocks from the drive signal (phase becomes random).

The unlocking pressure depends on the amplitude of the drive signal.

Can use this to measure the electric dipole moment of the microsphere (230 e·µm in this case).
Back to gravity: first generation attractors are patterned on Si-Au

Stationary electromagnetic shield

High ρ material: 
Au, \( \rho \approx 20 \text{ g/cm}^3 \)

Low ρ material: 
Si, \( \rho \approx 2 \text{ g/cm}^3 \)

\[ s_1 + t + s_2 \approx 5-30 \, \mu \text{m} \]
\[ t \approx 1 \, \mu \text{m} \]
\[ r_b = 5 \, \mu \text{m} \]
First generation attractor set, here shown w/o Au coating
Other attractor technologies are also being developed

Alternating droplets of low and high density liquids
High density liquid may be Hg or GallInStan

Example of microchannel in Si
New system, being commissioned

- Parabolic mirrors (no ghost beams/halos)
- Tilted vacuum windows (ditto)
- 2-inch optics (better Gaussian mode)
- More rigid mechanics
- More adjustable DOFs, many motorized DOFs (better alignment)
- Properly adjustable stationary shield
- Use rotation to average out electric dipole moment
- Gravity stage with larger throw (400µm instead of 80µm, signal further away from fundamental swing)
- 10x10 pixel photodiode (rudimentary, fast imaging)
- Better access to microsphere (convenience)
- Easily dismountable trap shields (ditto)
- Good quality metrology microscope
- More stable platform
10x10 Pixel, deep depletion photodiode array

- Custom devices, being tested now
- Soon we will be able to obtain crude images with interferometric information
- Need to design electronics
- FPGA-based real time processing
- Deep depletion:
  - ~100% QE at 1064nm
- Applicable to many other problems (LIGO feedbacks?)

Chris Kenney et al, SLAC
Expected sensitivity to short distance gravity

- $10^{-18}$ N/√Hz, $s = 7.5 \mu m$, $t = 10^5$ s
- $10^{-19}$ N/√Hz, $s = 2.5 \mu m$, $t = 10^6$ s

Excluded by experiments

Strength parameter, $|\alpha|$

Length scale, $\lambda$ [μm]

Yukawa messenger
Dilaton
heavy q moduli
 gluon modulus
Conclusions

• After >200 years of mechanical springs, we are developing a new technique to map the $1/R^2$ behavior of gravity at distances <50µm.
• It took a few years to catch up with the classic technique that has been perfected over many years, but we are getting there!
• Along the way we have discovered a wealth of tricks and applications to other areas of physics.
• We hope to have results relevant for gravity in the very near future.
• A discovery would be, of course, a game changer and would likely require confirmation from several groups, possibly using different techniques.
The cast

Also thank our “alter egos” at Yale:
Alec Emser, Adam Fine, Sumita Ghosh, Fernando Monteiro, David Moore, Cady van Assendelft